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Problem Sheet 5 73%

Particle Physics II

B4: Subatomic Physics

25 January, 2020

Question 5.1

Draw Feynman diagrams showing a significant decay mode of each of the following particles:

a) π^0 meson

b) π^+ meson

c) μ^-

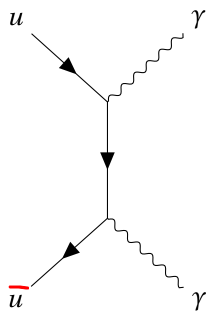
d) τ^- to a final state containing hadrons

e) K^0

f) top quark

What type of interaction is responsible for each of these decays? What is the reason why the decay you listed is more significant than others?

Solution. a) π^0 is a neutral meson and hence can decay electromagnetically into two photons. The Feynman diagram at tree level is given by (here π^0 is represented by $u\overline{u}$):



b) π^+ is charged, so it cannot decay electromagnetically. It is one of the lightest mesons, so it cannot undergo hadronic decay into other mesons. It has to decay weakly. By conservation of charge and leptonic family number, the decay product is $\ell^+ + \nu_\ell$, where ℓ^+ is the positron e^+ or the anti-muon μ^+ (the decay into τ^+ is not energetically possible). In fact the partial width of the decay

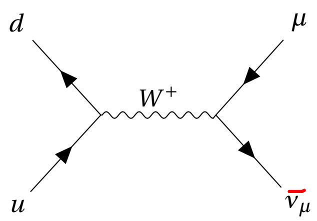
$$\Gamma_\ell \propto m_\ell^2(m_{\pi^+}^2 - m_\ell^2)^2$$

(The derivation is given in Section 9.4 of Griffiths' *Introduction to Elementary Particles*. I believe that it is beyond the syllabus of this course.)

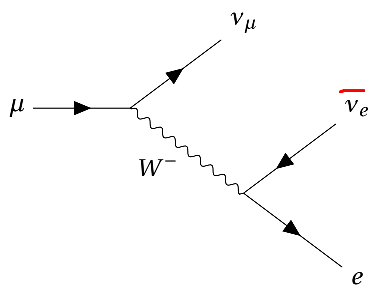
Since

$$\frac{m_e^2(m_{\pi^+}^2 - m_e^2)^2}{m_\mu^2(m_{\pi^+}^2 - m_\mu^2)^2} = 1.3 \times 10^{-4}$$

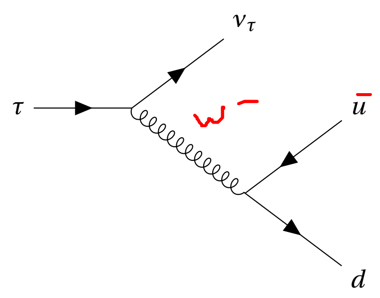
The dominating decay mode is $\pi^+ \to \mu^+ + \nu_\mu$, which is a weak interaction. The Feynman diagram at tree level is given by



c) μ^- can only decay to the electron e^-, which is the only lepton lighter than μ^-. By conservation of leptonic family number, the decay process is given by $\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$. The Feynman diagram at tree level is given by

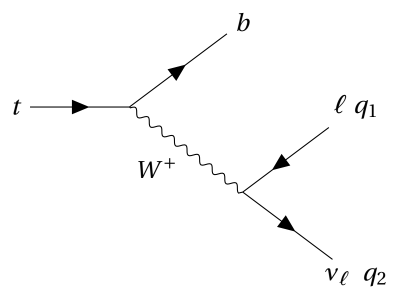


d) τ^- is sufficiently heavy to be able to decay into hadrons. A possible decay mode is $\tau^- \to \pi^- + \nu_\mu$, which is a weak interaction because the flavour is not conserved. The Feynman diagram at tree level is given by



e) The top quark t is very heavy so it decays faster than it can hadronise. The decay mode is $t \to b + W^+$, which is a weak interaction because the flavour is not conserved. The W boson later decays into two leptons or two quarks.

good!



[10/10]

□

Question 5.2

Draw the lowest-order Feynman diagrams for the process $e^+ + e^- \to \mu^+ + \mu^-$. Well above threshold, the expression for the total cross-section is

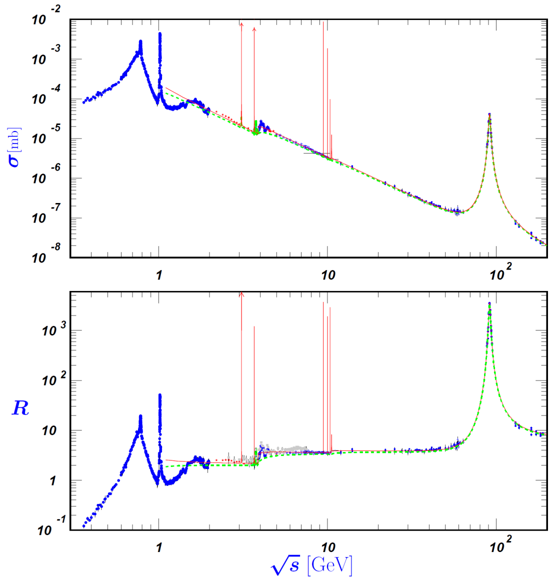
$$\sigma\left(e^+ + e^- \to \mu^+ + \mu^-\right) = \frac{4\pi}{3}\frac{(\alpha\hbar c)^2}{E^2} \quad \text{for} \quad E \ll M_Z c^2$$

where α is the electromagnetic fine-structure constant, M_Z is the mass of the Z^0 boson and E is the centre-of-mass energy.

a) Would you expect the same expression to be valid for the processes $e^+ + e^- \to e^+ + e^-$ and $e^+ + e^- \to \tau^+ + \tau^-$?

b) Draw leading order electromagnetic Feynman diagrams for the process $e^+ + e^- \to q + \overline{q}$. How do the vertex and propagator factors compare to before?

c) The figure below shows the cross-section $\sigma\left(e^+e^- \to \text{hadrons}\right)$ and the ratio of crosssections $R = \sigma\left(e^+e^- \to \text{hadrons}\right)/\sigma\left(e^+e^- \to \mu^+\mu^-\right)$ as a function of the centre-of-mass energy.

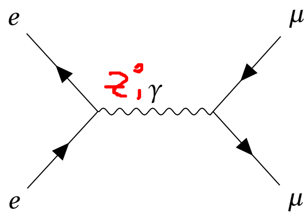


Considering the number of quarks that can be created at particular centre-of-mass energy, what values of R would you expect for centre-of-mass energy in the range $2\text{GeV} < \sqrt{s} < 20\text{GeV}$? How do your predictions match the data? How do these measurements support the existence of quark colour? What can you conclude on the structure of the quarks from these plots?

d) An e^+e^- collider operates at a centre-of-mass energy of 30GeV with a luminosity of $3\times 10^{35}\ \text{m}^{-2}\ \text{s}^{-1}$. What is the rate of production of hadronic events at such a collider?

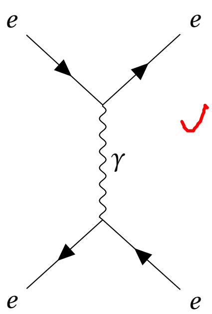
e) What is causing the sharp peaks in R at a centre-of-mass energy of about 3GeV and 10GeV, and the broader peak at about 100GeV?

Solution. Considering conservation of lepton family number at each vertex, the only possible tree-level diagram is given by



[1]

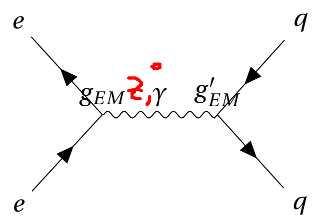
a) The expression is the same for the annihilation process $e^+ + e^- \to \tau^+ + \tau^-$. However, $e^+ + e^- \to e^+ + e^-$ is different, because the scattering process:



[2]

also contributes to the cross section at the tree level.

b) The Feynman diagram of $e^+ + e^- \to q + \overline{q}$ at tree level:



[1]

Since the coupling is electromagnetic, we expect to have the same vertex factor $g_{EM} = q/\sqrt{\epsilon_0 \hbar c}$ (proportional to the charge q) and the same propagator $1/P^{\mu}P_{\mu}$. However, notice that the quarks carry charges which are a fraction of e. Then the vertex factor at the production of the quark pair g_{EM} is either $1/3$ or $2/3$ of that of the electron-positron pair.

+ # of colours

c) Since the vertex factor is proportional to the charge, we have

$$R = 3\sum_i q_i^2$$

where q_i the charge of the quark produced (in the unit of e) and the sum is over all possible flavours of quarks produced. The factor 3 comes from the colour degree of freedom.

When 2 GeV $< \sqrt{s} <$ 3.5 GeV, the quarks produced are u, d, s. Therefore

$$R = 3\left(\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right) = 2$$

When 4 GeV $< \sqrt{s} <$ 10 GeV, the charm quark c can be produced. Then

$$R = 3\left(\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2\right) = \frac{10}{3}$$

When 11 GeV $< \sqrt{s} <$ 20 GeV, the beauty quark b can be produced. Then

$$R = 3\left(\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right) = \frac{11}{3}$$

[2]

The theory matches the plots nicely. If we had not count the degree of freedom contributed by the 3 different colours, the predicted R would be 1/3 of the measured value. This fact supports the existence of quark colour.

d) $\sigma = 0.35$ nb $R = 0.01\ s^{-1}$

e) The sharp peak at 3 GeV is the resonance of the meson $J/\Psi = c\bar{c}$ and at 10 GeV is the meson $\Upsilon = b\bar{b}$. The broader peak at 100 GeV is the resonance of the Z^0 boson. [1] □

[7/10]

Question 5.3

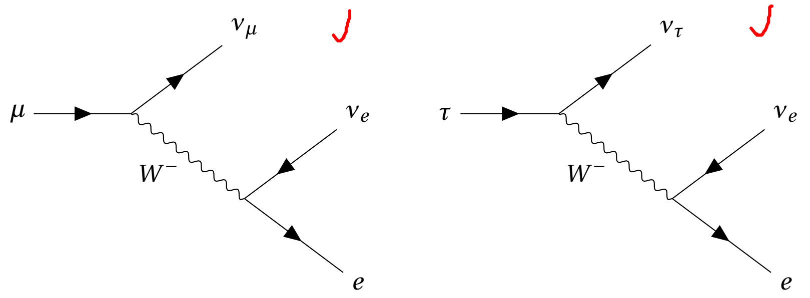
Draw Feynman diagrams for the decays of the muon and the tau lepton. Are hadronic decays possible? Making use of Sargent's rule explain why you would expect for the ratio

$$\frac{\Gamma\left(\tau^- \to e^- + \nu + \overline{\nu}\right)}{\Gamma\left(\mu^- \to e^- + \nu + \overline{\nu}\right)} = \left(\frac{m_\tau}{m_\mu}\right)^5$$

Test this prediction using the following data:

$$m_\tau = 1777.0\text{MeV}/c^2 \qquad \tau_\tau = 2.91 \times 10^{-13}\ \text{s}$$
$$m_\mu = 105.66\text{MeV}/c^2 \qquad \tau_\mu = 2.197 \times 10^{-6}\ \text{s}$$
$$BR\left(\tau^- \to e^- + \nu + \overline{\nu}\right) = 17.8\%$$

Solution. The decay modes of μ^- and τ^- are described in Question 1. μ^- has only leptonic decay because it does not have enough mass to decay into hadrons. τ^-, in fact, has many decay modes, and none of them is significantly dominating. The Feynman diagrams for the leptonic decay of μ^- and τ^- are given by [1]



Fermi's Theory shows that the partial width of the decay

$$\Gamma(\ell^- \to e^- + \nu_\ell + \overline{\nu}_e) \propto g_\ell^2 m_\ell^5$$

where the dependence of the fifth power of mass comes from the Sargent's rule. By leptonic universality, we expect that $g_\mu = g_\tau$. Therefore we have

$$\frac{\Gamma\left(\tau^- \to e^- + \nu + \overline{\nu}\right)}{\Gamma\left(\mu^- \to e^- + \nu + \overline{\nu}\right)} = \left(\frac{m_\tau}{m_\mu}\right)^5$$

From definition of the width we also have

$$\frac{\Gamma\left(\tau^- \to e^- + \nu + \overline{\nu}\right)}{\Gamma\left(\mu^- \to e^- + \nu + \overline{\nu}\right)} = \frac{\mathrm{Br}\left(\tau^- \to e^- + \nu + \overline{\nu}\right)\Gamma(\tau)}{\Gamma(\mu)} = \mathrm{Br}\left(\tau^- \to e^- + \nu + \overline{\nu}\right)\frac{\tau_\mu}{\tau_\tau}$$

From the given data, we have

$$\mathrm{Br}\left(\tau^- \to e^- + \nu + \overline{\nu}\right)\frac{\tau_\mu}{\tau_\tau} = 1.34387 \times 10^6, \qquad \left(\frac{m_\tau}{m_\mu}\right)^5 = 1.34550 \times 10^6$$

The prediction has relative error less than 0.12%. □

Question 5.4

a) Draw Feynman diagrams for the production of $W^\pm$ bosons being produced at a $p\overline{p}$ collider. If the W^+ boson is close to its Breit-Wigner peak, what possible decays may it have? (Which final states are kinematically accessible?)

b) What fraction of W^+ decays would you expect to produce positrons?

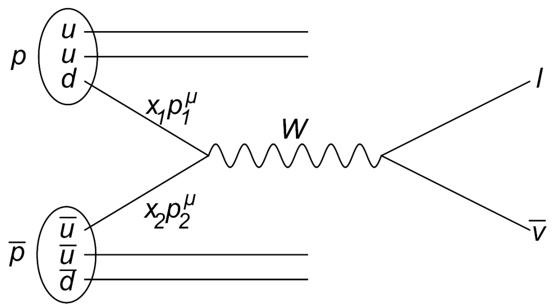
c) Suggest why the W was discovered in the leptonic rather than hadronic decay channels.

d) How could the outgoing (anti-)electron momentum be determined? How might the components of the neutrino momentum perpendicular to the beam be determined?

e) Draw diagrams for the decays $D^0 \to K^- + \pi^+$ and $D^0 \to K^- + e^+ + \nu_e$. Disregarding the differences in the 2-body and 3-body density of states factors, what do you expect for the relative rates of these decays?

f) How could W-bosons be produced at the LHC which collides protons with protons?

Solution. a) The diagram is hard to draw by TikZ. I borrow the diagram from the lecture notes:



The possible leptonic decay products of W^+ are $e^+ + \nu_e$, $\mu^+ + \nu_\mu$, $\tau^+ + \nu_\tau$. We note that $2m_b < m_W < m_t$, which means that W^+ can decay into any quarks other than the top quark t. The possible hadronic decay products of W^+ are $u\bar{d}$, $u\bar{s}$, $u\bar{b}$, $c\bar{d}$, $c\bar{s}$, and $c\bar{b}$. ✓ [3]

b) The fraction is 1/9. ✓ (I am not sure about the mechanism.) we'll discuss [1]

c) The identification of leptonic decays among the background are relatively easy compared to hadronic decays. ✓ [1]

d)

e)

f)

[5/10] □

Question 5.5

At the HERA collider 27GeV positrons collided with 920GeV protons.

a) By considering the de Broglie wavelength of the positron can you justify that these collisions can be considered to be due to positrons scattering off the quarks in the protons?

b) For these collisions draw **one** example of a Feynman diagram for **each** of the cases of weak charged-current, weak neutral-current and electromagnetic interaction.

c) Calculate the centre-of-mass energy of the quark-positron system assuming that the 4-momentum of the quark P_q can be represented as a fixed fraction x of the proton 4-momentum P_p, in the approximation where both particles are massless.

d) What is the highest-mass particle that can be produced in such a collision in the approximation that a quark carries about 1/3 of the proton momentum?

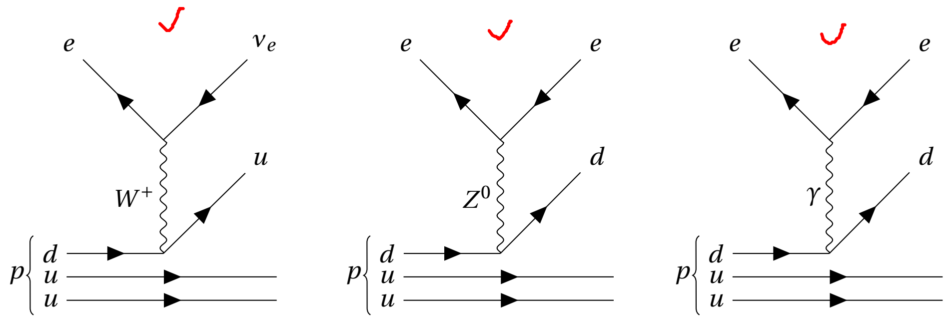
e) How does the propagator for the weak charged current and electromagnetic interactions vary with 4-momentum transfer P^2? Hence explain the fact that at low values of the momentum transfer it is found that the ratio of weak interactions to electromagnetic interactions is very small whereas at very high values it is found that the ratio is of the order of unity.

Solution. a) The positron is ultra-relativistic. It has momentum 27 GeV/c. The de Broglie wavelength is given by

$$\lambda = \frac{2\pi\hbar}{p} = 4.6 \times 10^{-17}\ \text{m} \quad ✓$$

The proton has diameter at the magnitude of 10^{-15} m, ✓ which is much larger than the de Broglie wavelength of the positron. Therefore we may consider the positrons interacting with the quarks inside the proton. ✓ [1]

b) The diagrams:



[4/8] [3]

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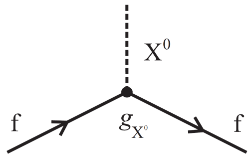
□

Question 5.6

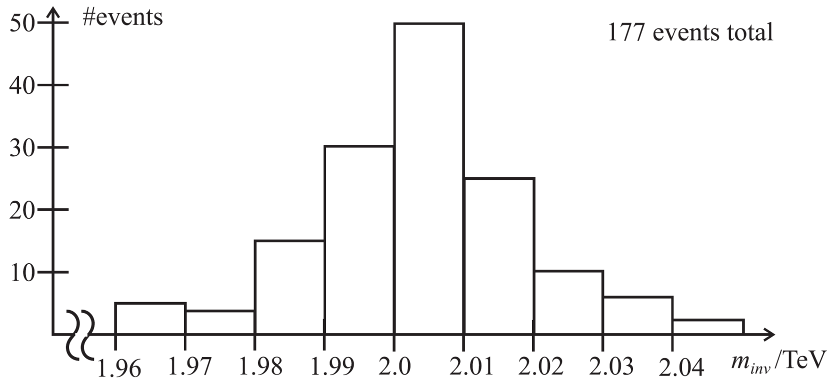
Let us assume that scientists at the pp collider LHC would discover a new particle X^0 of mass M_{X^0}. The only interactions it has are described by the Feynman diagram below, where g_{X^0} is a universal coupling constant valid for all fermions f.



Draw a parton level Feynman diagram of lowest order in g_{X^0} for the production of the X^0 at the LHC. Make sure to explain how any partons in your diagram connect to the valence quarks of the protons.

If the two constituents of the proton which ultimately produce an X^0 have momentum fraction x_1 and x_2 respectively and if the proton-proton centre-of-mass energy is $\sqrt{s}$ find an approximate relationship between $m_{X^0}, \sqrt{s}, x_1$ and x_2 that is independent of the proton mass. Clearly state any approximations you make.

The X^0 has been observed in its jet-jet, $\mu^+\mu^-$ and other final states. For an integrated luminosity of 2fb^{-1} the graph below shows the number of events $n_{jj}(m_{\text{inv}})$ per invariant mass interval in which the X^0 decays into a two-jet final state (background was subtracted) as a function of the invariant mass of its decay products m_{inv}.



Derive an expression for the ratio $N_{\mu^+\mu^-}/N_{jj}$ where $N_{\mu^+\mu^-}$ (N_{jj}) are the total number of events you expect to observe in the $\mu^+\mu^-$ (jet-jet) final states. Compare the shape and normalisation of the corresponding graph for $\mu^+\mu^-$ final states to the one shown above.

Write down the functional form of N_{jj} as a function of m_{inv} and the particle spins. Under which assumptions is this form accurate? You may ignore any normalisation constants. Explain the meaning of all terms in your functional form. Deduce the production cross-section $\sigma(pp \to X^0)$, the mass and the lifetime of the X^0 from the diagram. You may assume that that the detector which obtained this result is fully efficient and covers the full solid angle around the collision point.

Deduce the Baryon number of the X^0.

Solution. □